Power Output and Cost of a Wind Turbine

by David F. Williams, 29 August 2013

The cost-effectiveness of wind turbines for electrical power generation can be determined only by thorough analyses. However, the theoretical maximum electrical power output of wind turbines is readily derived from fundamental laws of physics. It's easy to calculate the mechanical kinetic power in air that moves through the circular area swept by a wind turbine's rotor. Consider a disc-shaped air mass (M) with diameter = D meters (equal to that of the wind turbine's rotor), with density = ρ kilograms per cubic meter, and with infinitesimal thickness = dx meters, moving with constant speed = v = dx/dt meters per second, as illustrated in Figure 1.

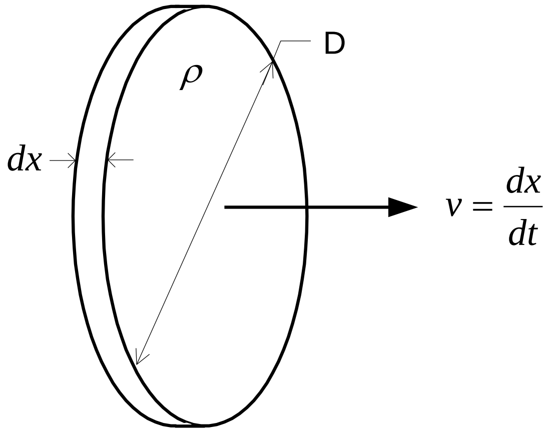

Figure 1. Infinitesimal Air Mass Moving at Constant Speed

This infinitesimal air mass has the infinitesimal mechanical kinetic energy:

$$dE_k = \frac{1}{2} M v^2 = \frac{1}{2}\left(\rho \, \frac{\pi D^2}{4} dx \right)\left(\frac{dx}{dt} \right)^2 \text{ watt sec} \qquad (1)$$

The infinitesimal air mass will move through the swept area of the wind turbine's rotor in the infinitesimal time interval = *dt* seconds. The time rate of change of energy observed at the wind turbine is equal to the infinitesimal energy in the moving air mass, divided by the infinitesimal time interval, *dt*. This time rate of change of energy is the mechanical power of the air mass:

$$P_m = \frac{dE_k}{dt} = \frac{1}{2}\left(\rho \, \frac{\pi D^2}{4} \frac{dx}{dt} \right)\left(\frac{dx}{dt} \right)^2 = \left(\rho \, \frac{\pi D^2}{8} \right)\left(\frac{dx}{dt} \right)^3 \text{ watt} \qquad (2)$$

With a constant wind speed, there is an infinite continuum of air-mass discs, each with infinitesimal thickness = *dx*, moving through the swept area of the wind turbine's rotor, and the sustained mechanical power in the wind passing through the swept area is given by equation (2). A German physicist, Albert Betz, proved that it is theoretically impossible for a wind turbine to capture more than 16/27 of this power. An ideal turbine would convert 100% of 16/27 P_m into electrical power. So, the maximum possible efficiency of a wind turbine ≅ 59.26%

At sea level, air density $\rho \cong 1.225$ kilograms per cubic meter. Figure 1 is a graph of 16/27 P_m (see equation 2 for P_m) as a function of wind speed (expressed in miles per hour, mph[1]) and turbine rotor diameter (meters, m). The output power is expressed in kilowatts (KW = watts/1000).

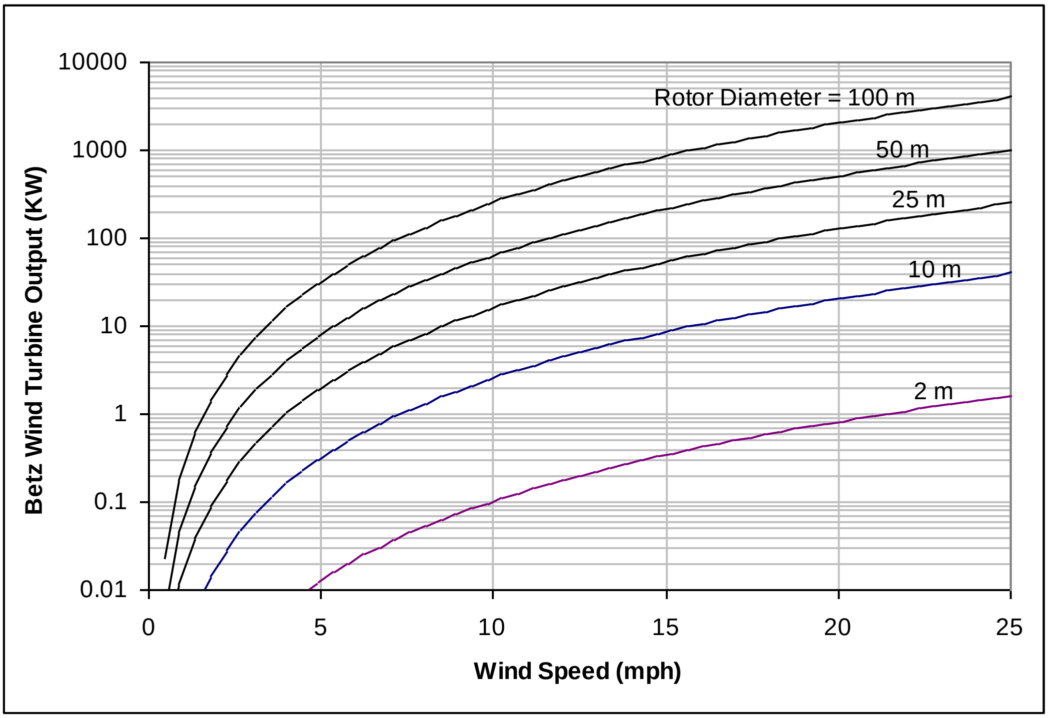

Figure 1. Power Output of an Ideal Wind Turbine (59.26% efficient) at Sea Level

The operating efficiency of an actual wind turbine depends on many factors. A wind turbine is designed to be most efficient at a particular wind speed, so any wind speed above or below this optimum will reduce the turbine's efficiency.

The average efficiency of a "good" wind turbine is approximately 35%[2]. Figure 2 illustrates the power output of such a wind turbine (transmission losses from the turbine to your home are not included). Superimposed on the curves in Figure 2 are the power requirements of a water heater and light bulb, plus the average power usage of a typical home.

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[1] One mile per hour equals 0.447 meters per second

[2] Michael A. Klemen, http://www.ndsu.nodak.edu/ndsu/klemen/index.htm.

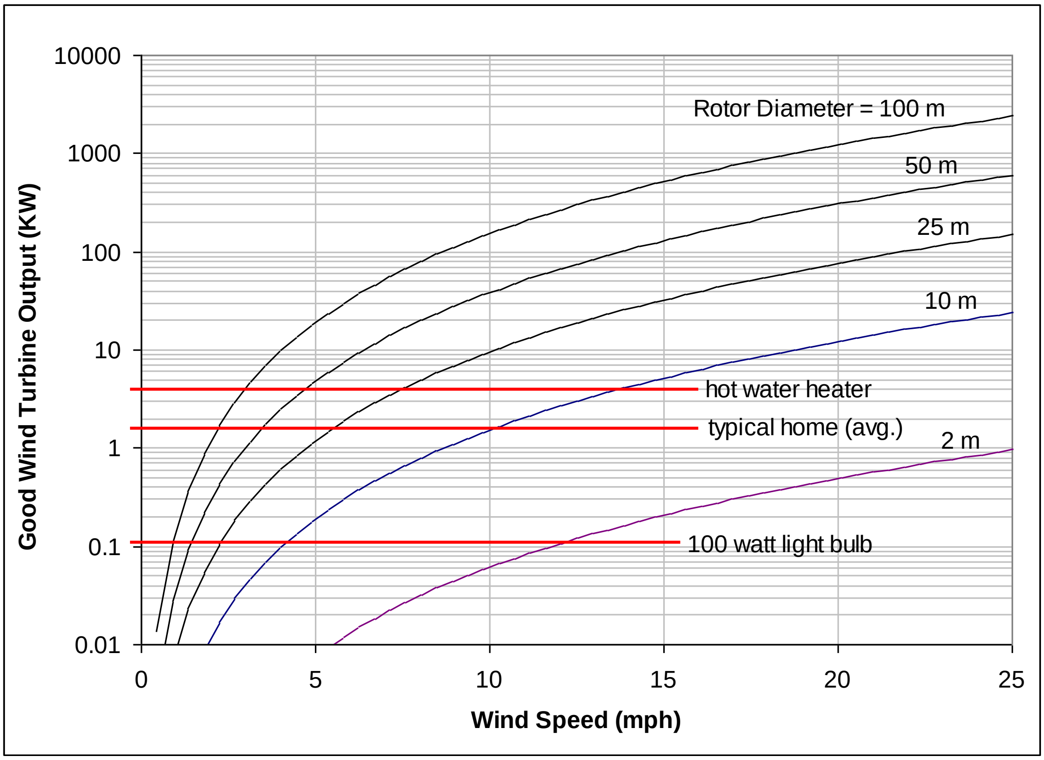

Figure 2. Power Output of a Good Wind Turbine (35% Efficient) at Sea Level

From Figure 2 it can be deduced that, with a constant 12.5 mph wind, you can power two 60 watt lamps with a good wind turbine having a rotor diameter of 2 meters. But, to operate a typical electric hot water heater at maximum power, the turbine rotor diameter would have to be over 10 meters.

The logarithmic scale of the plots in Figures 1 and 2 tends to "understate" the "cubed" effect of wind speed (see equation 2). If the wind speed doubles, the power output increases by a factor of 8. Conversely, if the wind speed halves, the power output decreases by a factor of 8. If your turbine has a rotor diameter of 2 meters, and the wind speed drops from 12.5 mph to 6.25 mph, you have to replace the two 60 watt lamps with one 15 watt lamp.

A huge turbine design, the GE 1.5sle, produces 1.1 megawatts (1100 kilowatts) of power at a wind speed of approximately 21.8 mph. The rotor diameter of this turbine is 77 meters. The maximum possible output (at the 59.26% Betz efficiency) for this wind speed and rotor diameter is 1560 kilowatts (see Figure 1). So, under these conditions this wind turbine operates at an efficiency that is 70.5% of the Betz maximum, converting 41.8% of the wind's mechanical kinetic power into electrical power. The height of the GE 1.5sle turbine's hub must be at least 65 m above the ground. As of March 2009, GE says that over 12,000 of these wind turbines are operating in 19 countries.

Using some of the cost data from a comprehensive analysis[3], combined with fundamental physics calculations and the reported mechanical-to-electrical efficiency of the GE 1.5sle wind turbine from another source, the cost per KWH at the output of the GE 1.5sle wind turbine was calculated (i.e., the "manufacturer's" cost). This cost per KWH is plotted in Figure 3 as a function of the number of years in operation and the wind speed. The electricity used in my house in Huntsville, Alabama, costs about $0.10 per KWH (retail). Considering the historic wind speeds in our area, and looking at Figure 3, which does not include the price increases in the path from manufacturer to the retailer, it doesn't take a rocket scientist to see that the wind turbine is not a viable source of electrical energy in our area.

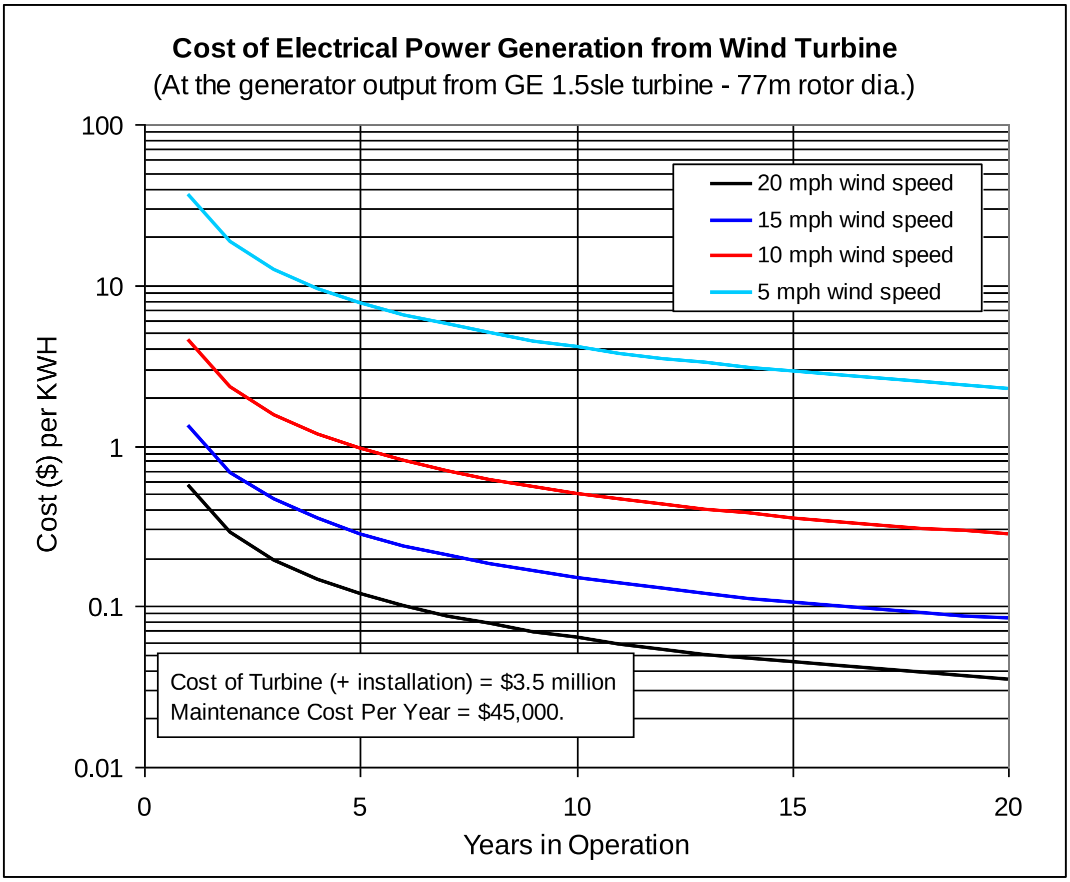

Figure 3. Cost of Electrical Power From GE 1.5sle Wind Turbine

It should be emphasized that the efficiency of an actual wind turbine varies with the wind speed. The data presented in this short report use the maximum efficiency of the turbine for all winds speeds. So, the data is optimistic.

[3] "Estimated Energy Production And Economics For A Large Wind Turbine Generator Installed At The Ipswich Municipal Light Department Site In Ipswich, Massachusetts", by William A. Vachon, W. A. Vachon & Associates, Inc., Manchester, MA 01944, March 2008